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Exact Local Supersymmetry, Absence of Superpartners, and Noncommutative Geometries*

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Abstract

It is pointed out that if we allow for the possibility of a multilayered universe, it is possible to maintain exact supersymmetry and arrange, in principle, for the vanishing of the cosmological constant. Superpartners of a known particle will then be associated with other layers of such a universe. A concrete model realizing this scenario is exhibited in 2+1 dimensions, and it is suggested that it may be realizable in 3+1 dimensions. The connection between this nonclassical geometry and noncommutative geometries is discussed.

1 Introduction

Supersymmetry provides a rich and elegant theoretical framework for treating fermions and bosons on the same footing. It has been the basis of many developments for over two decades ranging from supersymmetric quantum mechanics [1] and supersymmetric gauge theories [2] to superstring theories [3]. The usefulness of this concept as an approximate symmetry in atomic [4] and nuclear physics [5] is already indisputable. What is not yet clear is whether it is a symmetry of Nature at the most fundamental level, and if so in what form.

From a purely theoretical point of view, the rich mathematical structure of supersymmetry has been used to address a number of important unsolved physical problems. Among these are the gauge hierarchy problem, the cosmological constant

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problem, and the dark matter problem. Moreover, making use of such concepts as duality, holomorphicity, etc., supersymmetric gauge theories can be used to analyze the dynamics of gauge theories exactly [6]. This permits, among other things, a new approach to solving the longstanding strong coupling problem. Since the dynamical mechanisms made use of in these developments are standard to all gauge theories, it is hoped that they will also be applicable to non-supersymmetric gauge theories. It is thus clear that in the absence a competing framework general enough to address all of these problems, the optimism in the relevance of some form of supersymmetry in a fundamental theory is not unreasonable.

One serious drawback of supersymmetry as a fundamental symmetry is its lack of experimental support. Up to the presently available energies, there is no evidence for the existence of the superpartners of the known fundamental particles such as the electron and the photon. The standard interpretation of the absence of superpartners is to assume that supersymmetry is spontaneously broken and that, as a result, the superpartners acquire large masses, making them undetectable at currently accessible energies. It then follows that the absence of superpartners is only temporary and that experiments at a high enough energy scale will eventually lead to their discovery. Depending on the particular model, a typical lower bound for such a scale is of the order of Tev's. Unfortunately, there are no reliable upper bounds for this scale below the Planck scale.

From the experience with flavor symmetry and the manner in which different generations of quarks and leptons were predicted and discovered at higher and higher energies, it is generally believed that if supersymmetry plays a fundamental role, the above interpretation is the logical next step beyond the bosonic symmetries in particle physics. On the other hand, in a broader perspective, the consequences of the manner in which supersymmetry is broken are not confined to the particle physics sector. They will also have profound cosmological consequences. In particular, if supersymmetry is spontaneously broken in the usual way, one would have to look for a different mechanism to ensure the vanishing of the cosmological constant. So, if we look to supersymmetry as basis for the simultaneous solution of both the cosmological constant problem and the absence of superpartners, we appear to have reached an impasse.

A way out of this dilemma was suggested by Witten [7] based on how local supersymmetry is realized in 2+1 dimensions. There is also an alternative unconventional suggestion by my collaborators and me [8], which was again first encountered in connection with how local supersymmetry was realized in 2+1 dimensions. In the following sections, I will describe, in turn, these suggestions, how the alternative view was arrived at, its connection to noncommutative geometry, and some of its consequences.

2 Witten's Observation

As mentioned in the in previous section, the jest of Witten's observation is that in 2+1 dimensions the requirement of local supersymmetry provides a mechanism that, at least in principle, ensures the vanishing of the cosmological constant without leading to equality of masses for the supersymmetric partners [7]. The success of this mechanism depends crucially on the manner in which the states of nonzero energy (mass) are realized in 2+1 dimensions. To see this, we note that states of nonzero energy produce geometries which are asymptotically conical [9]. To have Fermi- Bose degeneracy in such a conical geometry, supersymmetry must be realized linearly, i.e., we must have asymptotic supersymmetric generators (supercharges) connecting fermionic and bosonic states of a supermultiplet. On the other hand, supercharges transform as Lorentz spinors so that their existence depends on whether the corresponding manifold allows the construction of spinors which are asymptotically covariantly constant. This cannot happen in an asymptotically conical geometry [10]. As a result, there will be no supercharges for constructing a linear representation of supersymmetry to which fermionic and bosonic states of nonzero mass could belong. Therefore, there will be no Fermi-Bose mass degeneracy. On the other hand, the geometry produced by the vacuum state which is a state of zero mass is not asymptotically conical, there are no restrictions on spinors, the vacuum remains supersymmetric, and the cosmological constant can be made to vanish.

If it were possible to implement this mechanism in $3+1$ dimensions in a realistic manner, it would significantly boost our near term confidence in the relevance of supersymmetry as a fundamental symmetry. The only obstacle on its way to full acceptance would then be its experimental confirmation.

3 An Alternative Proposal

In this section, I would like to present a point of view in which local supersymmetry is realized as a supermultiplet of space-times that I will refer to as a supersymmetric space-time. The geometry of such space-times are more complex than the familiar classical geometries and require the introduction of new concepts. It will be recalled that a classical metrical geometry is determined locally in terms of a differential line element or, equivalently, in terms of the components of a metric tensor. The supersymmetric space-time that we have in mind is an example of a nonclassical geometry which consists of the following elements : (i) The c-number line element is replaced with an "operator" line element. Equivalently, the components of the metric tensor are replaced with operators. (ii) These operators are constructed from the elements of an algebra. The particular algebras of interest in the present context are Lie and super Lie algebras. (iii) There is an associated Hilbert space on which the elements of the algebra act linearly. For a supersymmetric space-time, the corresponding Hilbert space is a supersymmetric multiplet realizing, say, the super Poincaré group.

The classical, long wave long wave length, limit of these nonclassical geometries can be determined by allowing the line element operator to act on the states of the associated Hilbert space. Then the diagonal elements may be replaced by the corresponding eigenvalues. As a result, for each state of the Hilbert space, the line element operator produces a "layer" of classical space-time, the number, n, of layers being equal to the dimension of the (super) multiplet. The off-diagonal elements of the line element operator provide the means of communication among various layers. Thus in this limit, a nonclassical geometry consisting of n layers of d-dimensional classical geometries may be viewed as a (d+1)-dimensional geometry in which the range of one of the dimensions is finite and discrete. As an example, consider an $N = 2$ supersymmetric space-time. It consists of four layers of d-dimensional space-times in which different layers are related to each other by supersymmetry transformations. We will see below a concrete realization of this nonclassical geometry in 2+1 dimensions.

Although the nonclassical geometry described above appears to be general and independent of the dimension d, it is conceivable that, like the mechanism suggested by Witten [7], it will only have 2+1 dimensional realizations. But for the moment let us assume that it will also have $3 + 1$ dimensional realizations and consider some of its predictions. Representing a particle by a Poincaré state, we can put such a particle and its superpartners in a supermultiplet consisting of these Poincaré states. Then the above nonclassical geometry, in its simplest form, suggests that the particle and its superpartners reside in different layers of the supersymmetric space-time. Since, in the simplest model, supersymmetry is the only means of communication between the layers, to have any hope of obtaining information about the superpartners, supersymmetry must remain exact. From this it follows that a particle and its superpartner(s), if they can be called that, must have the same mass and that the cosmological constant problem is, in principle, solvable.

An immediate difficulty with this picture which comes to mind is that there is no experimental evidence for superpartners of the same mass. In this respect, we must note that the experiments in question were all perceived and carried out under the assumption of a single layered Universe. So, one would not expect to obtain any information about the superpartners which "reside" in the other layers. Moreover, the very notion of a "superpartner" makes sense in a world of broken supersymmetry. In a superworld of exact supersymmetry, a particle and its superpartner(s) are different spin states of the same "superparticle". So one way of restating the lack of experimental evidence for the mass degenerate superpartners is to ask why it is that only one spin state of a superparticle appears in our experiments. A possible answer to this question is that a multilayered universe which emerges from a nonclassical geometry is very much like the many worlds picture necessary for an objective interpretation of quantum mechanics [11]. A superparticle in a multilayered universe is capable of being in any one of its spin states. In an experiment set up in any one layer, the wave function of the superparticle "collapses" into an eigenstate of spin consistent with that layer. In this sense, the other spin states are "confined." This makes the task

of obtaining information about superparticles highly non-trivial but not impossible. We must learn how quantum mechanics works in such a superworld. Needless to say, in the above discussion I have left out such intrinsic quantum mechanical effects as tunnelling, etc. I have also left out the possibility that for $N > 1$ there can be other off-diagonal operators connecting the layers even when supersymmetry is broken.

Finally, let me say a few words about a possible impact of a supersymmetric space-time picture on the dark matter problem. In a universe consisting of only one layer, there is excellent experimental support for the equality of the gravitational and inertial masses. In a multilayered universe, there is no á priori reason for equivalence principle to hold in its present form. So, it is plausible that the need for dark matter arises from the breakdown of the equivalence principle in a multilayered universe. For one thing, a superparticle of a given (inertial) mass contributes equally to the gravitational effects in every layer of this superworld.

4 Works on Noncommutative Geometries

The basic element of the nonclassical geometry described in the previous section is the introduction of an operator line element. The simplest way of viewing such an operator is to take the components of the metric tensor to be (noncommuting) operators. This statement is basis dependent, however, and a transformation to a different basis mixes the components of the metric tensor and the coordinates. Therefore, in the transformed basis the coordinates also become (noncommuting) operators. This means that we can view this nonclassical geometry as a form of a noncommutative geometry.

The subject of noncommutative geometry has appeared in theoretical physics in number of contexts. The most comprehensive among these is the work of Connes [12]. From a purely physical point of view, it has appeared in the works of Witten [13] and of 't Hooft [14]. It is also inherent in any quantum mechanical matrix model, or zero-brain formalism such as the work of reference [15] and the references cited therein. To my knowledge, no systematic study has been undertaken to see whether or not all of these works as well as our nonclassical geometry fall within the general formalism of Connes. The answer to this question is likely to accelerate the progress in this field.

5 Lessons from 2+1 Dimensions

To provide a concrete realization of the nonclassical geometry discussed in the previous sections, we now turn to the Chern Simons gauge theory of the super Poincaré group in 2+1 dimensions. It has been known for sometime that supergravity theories in 2+1 dimensions can be formulated as Chern Simons gauge theories of the corresponding supergroups [16-19]. In this and the following sections, I will explore the

physical properties of the emerging space-time when supersymmetric matter is coupled to these theories in a super Poincaré gauge invariant manner [20]. Let me begin with the simpler problem to the same aim, i.e., that of coupling matter to Poincaré Chern Simons gravity in a Poincaré gauge invariant manner. It has been shown [20] that the two-body problem for this theory is exactly solvable. One of the important features of this approach is that the concept of space-time is not a fundamental input but an output of the gauge theory.

The general form of the Chern Simons action in 2+1 dimensions given by

$$I_{cs} = \int_M \gamma_{bc} A^b \wedge (dA^c + \frac{1}{3} f^c_{\ de} A^d \wedge A^e) \tag{1}$$

where A^a are components of the Lie algebra valued connection

$$A = A^a G_a; \quad A^a = A^a_\mu dx^\mu \tag{2}$$

The quantities G^a are elements of the Lie algebra with structure constants f_{abc}. The quantities γ_{ab} are the components of a suitable non-degenerate metric on the Lie algebra [17]. For Poincaré algebra with elements P_a,J_a, a=0,1,2, the connection can be written as

$$A_\mu = e^a_\mu P_a + \omega^a_\mu J_a; \quad \mu = 0, 1, 2 \tag{3}$$

where e^a_μ and ω^a_μ are gauge fields of the Poincaré group. The manifold M in Eq. 1 is not to be identified with the metrical space-time.

Consider next the coupling of the Chern Simons action to matter. Any coupling via matter fields appear to break the local Poincaré gauge symmetry to its Lorentz subgroup, so that we are limited to matter coupling via sources. The Poincaré invariance of the Chern Simons gauge theory suggests that we introduce the notion of a particle or a source as an irreducible representation of the Poincaré group, in the same way as we do in particle physics in 3+1 dimensions. Then, its first Casimir operator $p^2 = m^2$ determines the mass of the source, and its second Casimir operator $W^2 = m^2 s^2$ its spin s. So, for sources of any spin, the coupling to the Chern Simons action can be achieved in terms of the action [20]

$$I = \int_C d\tau \ \eta_{ab} \left[p^a \partial_\tau q^b + t^\mu (p^a e^b_{\ \mu} + j^a \omega^b_{\ \mu})\right] + \lambda_1 (p^2 - m^2) + \lambda_2 (W^2 - m^2 s^2) \tag{4}$$

where $t^\mu = dx^\mu / d\tau$. It is clear from the action that the quantities p^a, and q^a are canonically conjugate to each other and satisfy Poisson brackets. For more than one source, we can add an action of this type for each one of them. In the presence of sources, the topology of the manifold M is modified, but the components of the field strength still vanish outside sources.

The problem of two sources coupled to the Chern Simons gravity can be solved by reducing it to an equivalent one-body problem [20]. This is done by taking full advantage of the topological features of the theory. In a topological gauge theory

all the gauge invariant observables can be expressed in terms of Wilson loops. This means that the Casimir invariants of a Poincaré state, which we identify as mass and spin, must be Wilson loops. Thus we can view our gauge invariant observables of this theory as the Casimir invariants of an equivalent one-body Poincaré state. Such a source is source endowed with two charges: a charge $\Pi^a = (\Pi^0, \vec{\Pi})$ and a charge $\Psi^a = (\Psi^0, \vec{\Psi})$, such that the Casimir invariants of the corresponding state are given, respectively, by $\Pi \cdot \Pi = H^2$ and $\Pi \cdot \Psi = HS$. We identify H and S as the mass and spin of the one-body source and wish to evaluate them in terms of Wilson loops of the two body system. For two sources with charges $(p_1^a,\ j_1^a)$ and $(p_2^a,\ j_2^a)$, respectively, the explicit evaluation of the Wilson loops were carried out in reference [20]. Here we quote the expression for H :

$$\cos\frac{H}{2} = \cos(\frac{m_1}{2})\cos(\frac{m_2}{2}) - \frac{p_1 \cdot p_2}{m_1 m_2}\sin(\frac{m_1}{2})\sin(\frac{m_2}{2}) \tag{5}$$

The Physical Space-Time

Let us now consider the structure of space-time which corresponds to this exact solution. Up to this point, we have constructed a Chern Simons gauge theory coupled to sources on R$\times\Sigma$ (x-space) which as we emphasized is metric independent and should not be identified with space-time. On the other hand,it is clear that the identification of quantities such as momenta and coordinates of physically realizable particles can only be made in a metrical space-time. So we must show how the notion of a metrical space-time emerges from this formalism and what our gauge invariant observables correspond to in such a space-time [21]. To this end, we recall that our two sources are characterized by charges (p_1^a, j_1^a) and (p_2^a, j_2^a) with the corresponding canonical coordinates q_1^a and q_2^a, respectively. Without loss of generality, let the first source be at rest at the origin, i. e. , $\vec{q}_1 = 0$. Then $\vec{q}_2 \equiv \vec{q}$ can be viewed as a relative coordinate. We parametrize $\vec{q}$ by its polar components: $\vec{q} = (r, \phi)$. By fixing $\vec{q}_1 = 0$, we have made a choice of gauge which fixes all the Poincaré gauge transformations except for the spatial rotations generated by J^0 and translations along q^0. To fix these, consider first the transformation

$$\vec{q'} = \left[\exp i\tau^0 J_0\right]\vec{q} \tag{6}$$

where

$$\tau^0 = (1 - \frac{H}{2\pi})\phi \equiv \alpha\phi = \phi' \tag{7}$$

Although H is a complicated function of the dynamical variables of the two sources, for the moment let us take it to be the numerical value of the exact Hamiltonian given by Eq. 5. Being an element of Poincaré group, this transformation leaves the Casimir invariants H and S unchanged. But the resulting vector, $\vec{q'}$, is no longer 2π periodic and satisfies the matching conditions for the coordinates on a cone characterized by

the deficit angle $\beta = H$. This can be seen by noting that the transformed coordinates $\vec{q'}$ acquire a phase under the rotation $\phi \rightarrow \phi + 2\pi$:

$$\vec{q'}(\phi + 2\pi) = [\exp{(2\pi - H)J_0}]\,\vec{q'}(\phi) \tag{8}$$

To completely fix the gauge, we must also fix translations along q^0. So, consider

$$q'^0 = q'^0(q^0, \phi') = q^0 - \frac{S\phi'}{2\pi\alpha} \tag{9}$$

where S is the numerical value of the second Casimir invariant of the Poincaré group.

It thus follows that the general reduction of the two-body problem to an equivalent one-body problem always leads, in a particular gauge, to the motion of the relative coordinate on a cone. We know from the analysis of metrical general relativity [9] that point sources generate conical space-times. For a single source, the deficit angle of the cone is determined by the energy (mass), E, of the source. We must therefore identify the quantity H with the total gravitational energy of the two body system. It generates a cone over which the relative coordinate of the reduced two body system moves. Despite their similarities, this cone should not be confused with the conical space of the test particle approximation. As is clear from Eq. 5, the deficit angle of our cone is determined by the Casimir invariant H which is a highly non-linear function of the masses and the momenta of the two sources. In terms of the gauge fixed variables, the expression for the line element takes the form

$$ds^2 = dq_0'^2 - dr^2 - r^2 d\phi'^2 \tag{10}$$

Or in terms of more familiar coordinates

$$ds^2 = (dq^0 - \frac{Sd\phi}{2\pi})^2 - dr^2 - \alpha^2 r^2 d\phi^2 \tag{11}$$

The coordinates in these equivalent expressions are related by Eqs. 6 and 9. Aside from any specific significance associated with the quantities H and S in this context, (see below), Eqs. 10 and 11 are standard expressions for the line element of a spinning cone [9].

It is thus clear that it is not the manifold R$\times\Sigma$ (x- space) but the q-space, M_q, from which the classical space-time is manufactured. Once the spatial part of q^a is identified with the cone, relativistic invariance requires that q^0 be identified with the "classical time". The quantity H characterizing this space-time also supplies [20], as it should, the boundary term which is necessary for the consistency of the canonical formalism in the metrical theory [22]. Since, as we have noted, H depends non-trivially on the momenta of the two sources, then, because the components of the metric tensor given by Eq. 11 also depend explicitly on the canonically conjugate variables, i.e., q_1 and q_2, these components will have non-vanishing Poisson Brackets with each other. Moreover, it follows from Eqs. 7 and 9 that in the form given

by Eq. 10 although the components of the metric are reduced to constants, the corresponding, primed, coordinates will have non-vanishing Poisson brackets. This suggests that, for consistency, the quantity S in Eq. 11, which is also a boundary term, should be replaced with $P.J/H$. This operator acts in the Hilbert space of the one-body Poincaré state. Thus we arrive at the "operator line element"

$$ds^2 = (dq^0 - \frac{P \cdot J d\phi}{2\pi H})^2 - dr^2 - \alpha^2 r^2 d\phi^2 \tag{12}$$

It is interesting to note that we can still write the line element operator in same form as that given by Eq. 10 if we define

$$q'^0 = q^0 - \frac{P \cdot J\phi}{2\pi H} \tag{13}$$

But then, as we have noted above, the coordinates in such a generalized geometry will no longer commute with each other.

In the classical large distance physics, the operators H and J may be safely replaced with their eigenvalues. But in a short distance quantum mechanical context, this geometrical non-commutativity may be significant and should not be ignored. As we will see in section 8, the operator interpretation of the line element will turn out to be crucial in describing the geometry of the supersymmetric space-time, even in the long wavelength limit.

6 Supersources as Supersymmetry Multiplets

This section is devoted to the description of supersources which are to be coupled to the Chern Simons action for the super Poincaré group. It will be recalled [20] that in the case of Poincaré gravity the sources(particles) can be viewed as irreducible representations of the Poincaré group. Similarly, we take a superparticle(supersource) to be an irreducible representation of the super Poincaré group. From this point of view, a superparticle is an irreducible supermultiplet consisting of several Poincaré states related to each other by the action of the supersymmetry generators. In the interest of explicitness, we will consider in detail the $N = 2$ super Poincaré group. The $N = 2$ super Poincaré algebra in 2+1 dimensions can be written as [23]

$$\begin{array}{lcl}
[J^a, J^b] = -i\epsilon^{abc} J_c & ; & [P^a, P^b] = 0 \\
[J^a, P^b] = -i\epsilon^{abc} P_c & ; & [P^a, Q_\alpha] = 0 \\
[J^a, Q_\alpha] = -(\sigma^a)_\alpha^{\ \beta} Q_\beta & ; & [P^a, Q'_\alpha] = 0 \\
[J^a, Q'_\alpha] = -(\sigma^a)_\alpha^{\ \beta} Q'_\beta & ; & \{Q_\alpha, Q_\beta\} = 0 \\
\{Q_\alpha, Q'_\beta\} = -\sigma^a_{\ \alpha\beta} P_a & ; & \{Q'_\alpha, Q'_\beta\} = 0 \\
a = 0, 1, 2 & ; & \alpha = 1, 2
\end{array} \tag{14}$$

The indices of the two component spinor charges Q_α and Q'_α are raised and lowered by the antisymmetric metric $\epsilon^{\alpha\beta}$ with $\epsilon^{12} = -\epsilon_{12} = 1$. the $SO(1,2)$ matrices σ^a satisfy the Clifford algebra

$$\{\sigma^a, \sigma^b\} = \frac{1}{2}\eta^{ab} \tag{15}$$

where η^{ab} is the Minkowski metric with signature $(+,-,-)$. We also have

$$\sigma^a{}_{\alpha\beta} = (\sigma^a)_\alpha^{\ \gamma}\epsilon_{\gamma\beta} \tag{16}$$

It is convenient to take the matrices σ^a to be

$$\sigma^0 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad ; \quad \sigma^1 = \frac{1}{2}\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad ; \quad \sigma^2 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{17}$$

The two Casimir operators of the super Poincaré group are given by

$$C_1 = P^2 = \eta^{ab}P_aP_b \tag{18}$$
$$C_2 = \eta^{ab}P_aJ_b + \epsilon^{\alpha\beta}Q'_\alpha Q_\beta \tag{19}$$

The first of these is the same as the Casimir operator of the Poincaré subgroup, so that its eigenvalues can be identified with the square of the mass of the superparticle. Since the Pauli-Lubanski operator (or its square) does not commute with supersymmetry transformations, it must be supplemented with the second term on the right hand side of Eq. 19 to obtain a super Poincaré invariant. We will designate its eigenvalues as mc_2.

Irreducible representations of the $N = 2$ super Poincaré group in 2+1 dimensions can be constructed along the same lines as those in 3+1 dimensions [24]. For massive states, without loss of generality we can work in a frame in which the supermultiplet is at rest. Then the non-vanishing anti-commutators of the superalgebra simplify to

$$\{Q_1, Q'_2\} = \{Q_2, Q'_1\} = \frac{m}{2} \tag{20}$$

Thus Q_α and Q'_α, $\alpha = 1, 2$, form a Clifford algebra. We define a Clifford vacuum state , $|\Omega>$ by the requirement

$$Q_\alpha|\Omega> = 0 \quad ; \quad \alpha = 1, 2 \tag{21}$$

It is easy to verify that such a state exists within every supermultiplet and that it is an eigenstate of C_1 and C_2:

$$C_1|\Omega> = m^2|\Omega> \tag{22}$$
$$C_2|\Omega> = mc_2|\Omega> \tag{23}$$

From the definition of the Clifford vacuum state in the rest frame of the superparticle, it follows that

$$\begin{aligned} C_2|\Omega> &= P\cdot J|\Omega> \\ &= ms^0|\Omega> \\ &= ms|\Omega> \end{aligned} \tag{24}$$

where we identify the eigenvalue, s, of the operator s^0 with the spin of the state $|\Omega>$. So, the Clifford vacuum state is a Poincaré state with mass m and spin s:

$$|\Omega>=|m,s> \tag{25}$$

Consider, next, the states

$$|\Omega_1> = Q'_1|\Omega>, \tag{26}$$
$$|\Omega_2> = Q'_2|\Omega> \tag{27}$$

and

$$|\Omega_{12}>=Q'_1Q'_2|\Omega> \tag{28}$$

It is easy to verify that

$$s^0|\Omega_1> = (s-\frac{1}{2})|\Omega_1> \tag{29}$$
$$s^0|\Omega_2> = (s+\frac{1}{2})|\Omega_1> \tag{30}$$
$$s^0|\Omega_{12}> = s|\Omega_{12}> \tag{31}$$

These three Poincaré states together with the Clifford vacuum state form an Irreducible supermultiplet of $N=2$ super Poincaré group in 2+1 dimensions, which we call a superparticle. Each supermultiplet is distinguished by its mass m and the eigenvalue $c_2=s$, where s is the spin of the Clifford vacuum state. The spins of the states within a supermultiplet are fixed once the value of c_2 is specified. For example, for $c_2=\frac{1}{2}$, the resulting $N=2$ supermultiplet is a vector multiplet consisting of a spin zero, two spin 1/2, and one spin one Poincaré states.

7 Exact Solution of the Two-Superbody Problem

It has been pointed out recently that the two-superbody problem in $N=2$ Chern Simons supergravity is exactly solvable [23]. It was in the process of giving a physical interpretation to this solution that the departure from classical to nonclassical geometry became unavoidable [8]. I will briefly sketch the two superbody problem below and go over the supersymmetric space-time which emerges from it in the next section. As in Section 5, we begin with the general form of the Chern Simons action in 2+1

dimensions given by Eq. 1. In the present case, the quantities A^B are the components of the Lie superalgebra valued connection which for $N = 2$ super Poincaré algebra can be written as

$$A_\mu = e_\mu^{\ a} P_a + \omega_\mu^{\ a} J_a + \chi_\mu^{\ \alpha} Q_\alpha + \xi_\mu^{\ \alpha} Q'_\alpha \tag{32}$$

Then the covariant derivative is

$$D_\mu = \partial_\mu + i A_\mu \tag{33}$$

Then, just as in Poincaré gravity, the Chern Simons action for the super Poincaré group can be written as

$$\begin{aligned} I_{cs} &= \frac{1}{2} \int_M \{\eta_{bc}[e^b \wedge (2d\omega^c + \epsilon^c_{\ da} \omega^d \wedge \omega^a)] \\ &\quad - \epsilon_{\alpha\beta}[\chi^\alpha \wedge (d - i\sigma_a \omega^a)\psi^\beta + \psi^\alpha \wedge (d - i\sigma_a \omega^a)\chi^\beta]\} \end{aligned} \tag{34}$$

As in the case of Poincaré gravity, the manifold M is specified by its topology and is not to be identified with space-time which will emerge (see below) as an output of this gauge theory.

To couple (super)sources to this Chern Simons theory, we proceed in a manner similar to the way sources were coupled to the Poincaré Chern Simons theory. From the discussion of the supermultiplets given in section 6, we conclude that the logical candidates for our supersources are the irreducible representations of the $N = 2$ super Poincaré group. Then each supersource can be coupled to the $N = 2$ Chern Simons supergravity by an action of the form [23]

$$\begin{aligned} I_s = \int_C d\tau \{ & p_a \partial_\tau q^a - \epsilon_{\alpha\beta} p^\alpha \partial_\tau q^\beta - t^\mu (e^a_\mu p_a + \omega^a_\mu j_a - i\epsilon_{\alpha\beta} \chi^\alpha_\mu p^\beta \\ & + (\sigma \cdot p)_{\alpha\beta} \xi^\alpha_\mu q^\beta) + \lambda_1 (p^2 - m^2) + \lambda_2 (c_2 - s)\} \end{aligned} \tag{35}$$

where τ is an invariant parameter along the trajectory C. Also, mc_2 is an eigenvalue of the second Casimir operator of the super Poincaré group, and s is the spin of the Clifford vacuum state of the supermultiplet. The choice of the constraint multiplying λ_2 is crucial in relating the eigenvalue of the second Casimir invariant, c_2, of the superalgebra to the spin content of a supermultiplet. For more than one source, one can add an action of this type for each source. In the presence of supersources the topology of the manifold is modified. But the field strengths still vanish outside supersources, and the theory is locally trivial.

It was shown in reference [23] that the exact gauge invariant observables of the two-superbody system can be obtained in terms of Wilson loops. They may be viewed as the Casimir invariants of an equivalent one-superbody state, similar to the equivalent one-body state of Chern Simons gravity. We will refer to these invariants as H and C_2. As we have seen above, their eigenvalues determine mass(energy) and spin(angular momentum) content the supermultiplet. They constitute the asymptotic observables of the two-superbody system and were given in references [23]. Here we note that the expression for the invariant H is identical to the corresponding invariant for its Poincaré subgroup given by Eq. 5.

8 The Physical Space-Time

Having discussed the gauge invariant observables of the exact two-superbody system, we now turn to the structure of the corresponding space-time. We take our clue from the space-time structure which emerged in section 5 from the dynamics of the two-body system in Poincaré Chern Simons gravity. In the supersymmetric case, the situation is somewhat more complicated. To see why, we note that in both cases we can associate our gauge invariant observables to a reduced one-(super)body state. In the pure gravity case, such a state is a single Poincaré state. In the supersymmetric case it is a supermultiplet consisting of several (four for $N = 2$) Poincaré states. As we saw in section 5, in the case of Poincaré Chern Simons theory, the structure of the emerging space-time and its asymptotic observables are completely determined by the two (gauge invariant) Casimir invariants of the reduced one-body Poincaré state. To see how this picture generalizes for the two-superbody system, we recall that our two supersources are characterized by charges (p_1^A, j_1^A) and (p_2^A, j_2^A) with the corresponding canonical coordinates q_1^A and q_2^A, respectively. Without loss of generality, let the first supersource be at rest at the origin, i. e. , $\vec{q}_1 = 0$. Then $\vec{q}_2 \equiv \vec{q}$ can be viewed as a relative coordinate. As in pure gravity, we parametrize $\vec{q}$ by its polar components: $\vec{q} = (r, \phi)$. By fixing $\vec{q}_1 = 0$, we have again made a choice of gauge which fixes all the $N = 2$ super Poincaré gauge transformations except for the rotations generated by J^0 and translations along q^0. To fix these, consider first the same transformation as that specified by Eqs. 8 and 9. Being an element of $N = 2$ super Poincaré group, this transformation leaves the Casimir invariants H and C_2 unchanged. But again the $\vec{q'}$ is no longer 2π periodic and satisfies the matching conditions for the coordinates on a cone characterized by the deficit angle $\beta = H$.

Up to this point, everything looks the same as in Poincaré gravity discussed in Section 5. However, differences begin to appear when we try to gauge fix the translations along q^0. It will be recalled from our discussion of supersources that an $N = 2$ supermultiplet at rest with Casimir invariants H and C_2 consists of four Poincaré states. Writing the eigenvalues of C_2 as Hc for the Clifford vacuum, these four states will have the following spin eigenvalues :

$$\begin{aligned} P \cdot J|H, c, s_1 > &= H(c - \frac{1}{2})|H, c_2, s_1 > && (36)\\ P \cdot J|H, c, s_2 > &= Hc|H, c, s_2 > && (37)\\ P \cdot J|H, c, s_3 > &= Hc|H, c, s_3 > && (38)\\ P \cdot J|H, c, s_4 > &= H(c + \frac{1}{2})|H, c, s_4 > && (39) \end{aligned}$$

In the case of Poincaré Chern Simons gravity, it was possible to also fix the gauge in q^0 direction by the transformation given by Eq. 9 which involved the spin of the Poincaré state. Clearly, this is no longer possible for a supermultiplet consisting of Poincaré states of different spin. This makes it impossible for a single c-number line element

of the form given by Eqs. 10 and 11 to describe all the spin states of our equivalent one-superbody multiplet even in the case of classical large distance gravity. So, to describe all the spin states corresponding to our gauge invariant observables H and C_2, we must generalize the usual notion of a c-number line element to the "operator line element" given by Eq. 12, which now acts on the Poincaré states making up the supermultiplet. When this operator line element acts on a state of a supermultiplet, we can replace, at least for large distance physics, the operator $P.J/H$ with the spin eigenvalue of that state and hence specify the corresponding c-number space-time. It therefore follows that the description of all the spin states of the equivalent one-body supermultiplet requires a multiplet of space-times equal in number to the dimension of the supermultiplet (four for $N = 2$). With $k = 1, .., 4$, the line elements for the members of this space-time multiplet are given by

$$ds_k^2 = (dq^0 - \frac{s_k d\phi}{2\pi})^2 - dr^2 - \alpha^2 r^2 d\phi^2 \tag{40}$$

The line element operator in Eq. 12 is not invariant under supersymmetry transformations, and it transforms in the same way as the Poincaré states within a supermultiplet. In other words, for $k = 1, .., 4$ the line elements in Eq. 40 form an irreducible representation of the $N = 2$ supersymmetry and are completely determined by the asymptotic observables H and C_2. Thus, the metrical description of the two-super- body system coupled to the super Poincaré Chern Simons action requires not just one but a supermultiplet of space-times. The supersymmetry generators act as ladder operators relating different layers of this nonclassical geometry. In its simplest form such as in the classical large distance regime, this supersymmetric space-time may be viewed as an ordinary space-time with an additional finite discrete dimension.

We have thus verified that the supersymmetric space-time which emerges from the exact solution of the two-superbody in 2+1 dimensions is, in all details, a realization of the nonclassical geometry discussed in section 3.

9 Concluding Remarks

Like Witten's suggestion described in section 2, it might be thought that the interesting applications of the nonclassical geometry described in this work are confined to 2+1 dimensions. This may well turn out to be the case. However, it is not difficult to conceive of $3 + 1$ dimensional realizations of this geometry, which may or may not be interesting. For example, noting the correspondence between point-like sources in 2+1 dimensions and infinite line sources in $3 + 1$ dimensions, one can extend the supersymmetric space-time discussed in the previous section to $3 + 1$ dimensions by simply adding a dz^2 term to each line element in Eq. 40. Work is in progress to see how one can detect experimentally, and possibly rule out, the multilayer effects of a supersymmetric space-time. For one thing, this may also be a way of testing the many worlds picture of quantum mechanics. Much remains to be clarified.

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References

1. E. Witten, *Nucl. Phys.* **B188** (1981) 513.
2. J. Wess and B. Zumino, *Nucl. Phys.* **B70** (1974) 39.
3. M.B. Green, J.H. Schwarz, E. Witten, *Superstrings*, **Vol. I, Vol. II**, Cambridge University Press, 1985.
4. V.A. Kostelecky and M.M. Nieto, *Phys. Rev.* **A32** (1985) 1293.
5. F. Iyacello, *Phys. Rev. Lett.* **44** (1980) 772.
6. N. Seiberg and E. Witten, *Nucl. Phys.* **B426** (1994) 19, **431** (1994) 484.
7. E. Witten, *Int. Jour. Mod. Phys.* **A10** (1995) 1247.
8. A preliminary version of our results were first reported at *The International Conference on Seventy Years of Quantum Mechanics and Modern Trends in Theoretical Physics*, Calcutta, India 1/29- 2/2/1996, to appear in the Proceedings, ed. P. Bandyopadhyay; Sunme Kim and Freydoon Mansouri, e-print gr-qc/9609037, *Phys. Lett.* **B397** (1997) 81; F. Ardalan, S. Kim, F. Mansouri, *Int. Jour. Mod. Phys.* **A12** (1997) 1183
9. S.Deser, R.Jackiw, G. 't Hooft, *Ann. of Phys.* (N.Y.) **152** (1984) 220; S. Giddings, J. Abbott and K. Kuchar, *Gen. Rel. Grav.* **16**(1984) 751; J.R. Gott and M. Alpert, *Gen. Rel. Grav.* **16** (1984) 751.
10. M. Henneaux, *Phys. Rev.* **D29** (1984) 2766
11. H. Everett, III, in *The Many Worlds Interpretation of Quantum Mechanics*, ed. by B.S. DeWitt and N. Graham, Princeton University Press, 1973.
12. A. Connes, *Non-commutative Geometry*, Academic Press, 1994
13. E. Witten, *Nucl. Phys.* **B460** (1995) 335.
14. G. 't Hooft, Utrecht preprint THU-96/02, e-print gr-qc/9601014
15. T. Banks, W. Fischler, S.H. Shenker, L. Susskind, e-print hep-th/9610043.

16. A. Achucarro and P.K. Townsend, *Phys. Lett* **B180** (1986) 89

17. E. Witten, *Nucl. Phys.* **B311** (1988) 46 and **B323** (1989) 113

18. K. Koehler, F. Mansouri, C. Vaz, L. Witten, *Mod. Phys. Lett.* **A5**(1990) 935

19. K. Koehler, F. Mansouri, C. Vaz, L. Witten, *Nucl. Phys.* **B341** (1990) 167 and**B348** (1990) 373

20. F. Mansouri and M.K. Falbo-Kenkel, *Mod. Phys. Lett.*; *Jour. Math. Phys.* **A8** (1993) 2503; F. Mansouri, *Comm. Theo. Phys.* **4** (1995) 191.

21. F. Mansouri in *Proceedings of Twenty Third Coral Gables Conference*, ed. B. Kursunoglu, S. Mintz, A. Perlmutter, Plenum Press, 1995

22. T. Regge and C. Teitelboim, *Ann. Phys.* **88** (1974) 236

23. Sunme Kim and F. Mansouri, *Phys. Lett.* **B 372**(1996) 72

24. A. Salam and J. Strathdee, *Forts. Phys.* 26 (1978) 57; P.G.O. Freund, *Introduction to Supersymmetry*, Cambridge University Press, 1986; J. Wess and J. Bagger, *Supersymmetry and Supergravity*, second edition, Princeton University press, 1992